MATH 476 – College Geometry

Solutions to Homework Assignment 6

1. **Section 4.3:** 1, 2, 3, 14, 18

   1. See back of book.
   2. $\triangle ABC \sim \triangle ZXY$ by SSS similarity.
   3. See back of book.

   14. If the outermost square has side length x, then the triangle in the upper left corner of that square is a 45-45-90 triangle with leg $\frac{1}{2}x$, so the length of its hypotenuse is $\frac{\sqrt{2}}{2}x$. The same ratio applies going from the middle square to the small square, so the side length of the small square is $\left(\frac{\sqrt{2}}{2}\right)^2 x = \frac{1}{2}$. Thus, the ratio is $\frac{1}{2}$.

   18. (a) If $\angle A$ is acute, then $\triangle ABE \sim \triangle ACF$ (note typo) by AA since they both have a right angle and they share $\angle A$. If $\angle A$ is obtuse, the same criterion applies, but it applies with supplements to $\angle A$ rather than $\angle A$ itself. (If we have a right triangle, then $A = E = F$.)

       (b) Consider the third figure from Exercise 17. Take $\overline{BC}$ as the diameter of a circle. Since m$\angle BEC = 90$ and m$\angle CFB = 90$, both B and F lie on this circle. Note that $\angle FCB$ and $\angle FEB$ open onto the same angle, so $\angle FCB \cong \angle FEB$. Similarly, $\angle FBE \cong \angle FCE$. Now $\angle AEB$ (a right angle) is exterior to $\triangle BEC$, so 90 =m$\angle AEF$+$m\angle FEB$ =m$\angle AEB$ =m$\angle ECB$+m$\angle EBC$ =m$\angle ECF$+m$\angle FCB$+m$\angle EBC$ Thus m$\angle AEF$+m$\angle FCB$ =m$\angle ECF$+m$\angle FCB$+m$\angle EBC$ by substitution, so m$\angle AEF$ =m$\angle ECF$+m$\angle EBC$ =m$\angle FBE$+m$\angle EBC$ =m$\angle ABC$. Thus by Exercise 17, $\triangle AFE \sim \triangle ACB$.

2. **Section 4.5:** 1, 3, 12, 15, 22

   1. See back of book.
   3. See back of book.

   12. (a) m$\angle 2 = 180 - 2\theta$ =m$\angle 1$ (by the Inscribed Angle Theorem). The complement of minor arc $\widehat{DB}$ is a semicircle along with minor arc $\widehat{BC}$. Thus, m$\angle 3 = 180 - (90+$m$\angle 1) = 2\theta - 90$. Finally, m$\angle 4 = 180-$m$\angle 3 - \theta = 270 - 3\theta$.

       (b) If m$\angle 3 = 60$, we get $\theta = 75$.

   15. See back of book.

   22. (a) This follows immediately from the Pythagorean Theorem.

       (b) Let $x = PO$. Then we wish to know for which values of x do we get $x^2 - r^2 = k$. This will occur if $x^2 = k + r^2$, or $x = \sqrt{k + r^2}$. Thus, the distance from P to O must be a constant, so the set of points desired is another circle centered at O.

3. **Section 4.8:** 4, 10

   4. We have $\frac{AF}{FB}\frac{BD}{DC}\frac{CE}{EA} = \frac{AX}{XB}\frac{BD}{DC}\frac{CE}{EA}$. Thus $\frac{AF}{FB} = \frac{AX}{XB}$. Assigning coordinates a, b, f, and x in the obvious way, we find $\frac{f-a}{b-f} = \frac{x-a}{b-x}$, so $fb - fx - ab + ax = bx - ab - fx + fa$. Thus $fb + ax = bx + fa$, giving $f(b-a) = x(b-a)$. Therefore, $f = x$, so $F = X$.

   10. See the page in my box. (Please don't take it, though!)